5/3/18

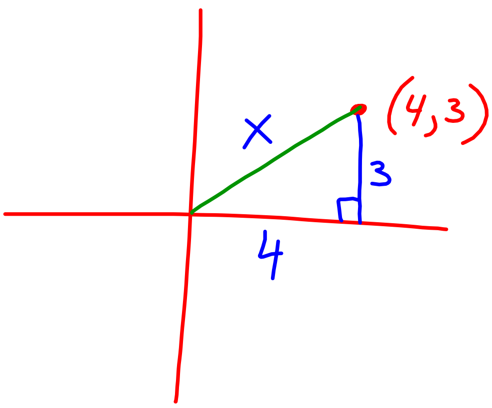
"If it matters to you, you will find a way. If it doesnt, you will find an excuse." -Unknown

HW: Test 2 Wednesday 5/23

AIM: What is the Length of a Curve?

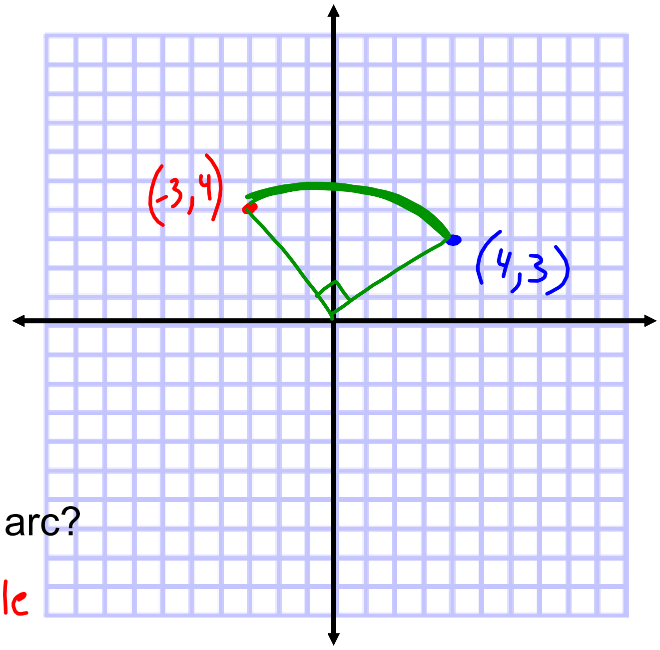
Warm Up:

1) How far is (4, 3) from the origin?



$$4^2 + 3^2 = x^2$$
$$16 + 9 = x^2$$
$$25 = x^2$$
$$5 = x$$

2) If we rotate (4, 3) 90 degrees (counterclockwise) about the origin, what are the new coordinates?



3) What is the length of the green arc?

The arc is $\frac{1}{4}$ of the circle

$C = \pi d$

$C = \pi 2r$

$arc = \frac{\pi 2r}{4} = \frac{\pi 2(5)}{4} = \frac{10\pi}{4} \approx \boxed{7.85 \text{ units}}$

(*) The length of a curve $y=f(x)$ from $x=a$ to $x=b$ is:

$$\text{Length} = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

4) Verify that 7.85 is the length of #3

$\underset{x=4}{(4,3)}$ and $\underset{x=-3}{(-3,4)}$

The function is part of the circle with center $(0,0)$ radius $=5$

$$x^2+y^2=25$$

$$y^2=25-x^2$$

$$y=\pm\sqrt{25-x^2}$$

$$\text{length} = \int_{-3}^{4} \sqrt{1+(f'(x))^2}\,dx$$

arc is part of $y=\sqrt{25-x^2}$

$$f(x)=\sqrt{25-x^2}$$

$$f(x)=(25-x^2)^{\frac{1}{2}}$$

$$f'(x)=\frac{1}{2}(25-x^2)^{-\frac{1}{2}}\cdot(-2x)$$

$$f'(x)=\frac{-2x}{2\sqrt{25-x^2}}$$

$$f'(x)=\frac{-x}{\sqrt{25-x^2}}$$

$$\text{Length} = \int_{-3}^{4} \sqrt{1+\left(\frac{-x}{\sqrt{25-x^2}}\right)^2}\,dx$$

NORMAL FLOAT AUTO REAL RADIAN MP

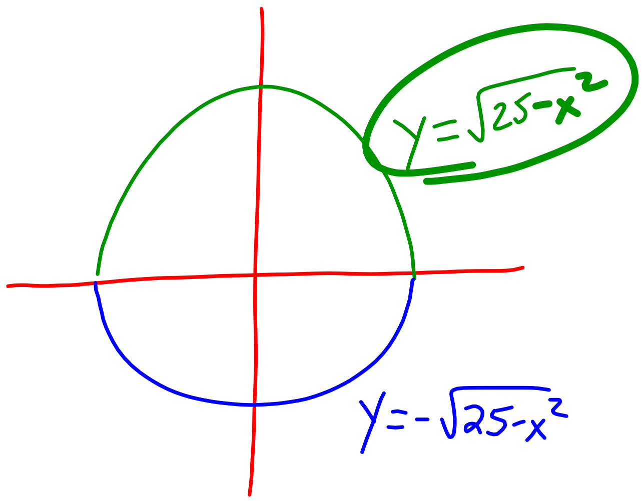
$$\int_{-3}^{4}\left(\sqrt{1+\left(\frac{-X}{\sqrt{25-XX}}\right)^2}\right)dX$$

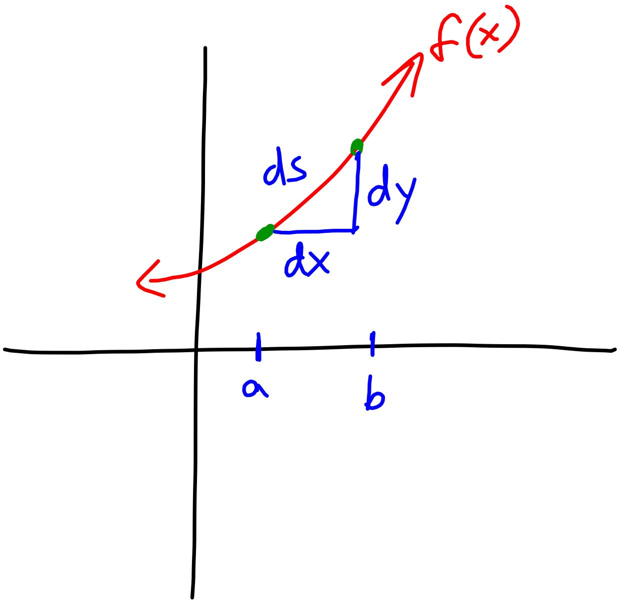
7.853981634

$$\int_{-3}^{4}\left(\sqrt{1+\left(\frac{d}{dX}\left(\sqrt{25-XX}\right)\Big|_{X=X}\right)^2}\right)dX$$

7.853981727

✓


y = √25-x²
y = -√25-x²



$$(ds)^2 = (dx)^2 + (dy)^2$$

$$ds = \sqrt{(dx)^2 + (dy)^2}$$

$$ds = \int_a^b \sqrt{(dx)^2 + (dy)^2}$$

$$= \int_a^b \sqrt{(dx)^2 \left(1 + \left(\frac{dy}{dx}\right)^2\right.}$$

$$= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

derivative